Relativistic space-time geometry and kinematics of uniformly moving objects from Hamilton's Principle

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Abstract

Hamilton's Principle is used to derive, from a covariant Lagrangian, relativistic equations describing space-time geometry and kinematics of a uniformly-moving ponderable object. The time transformation is a universal position-independent time dilation relation without any 'relativity of simultaneity' effect. An appendix shows how spurious 'length contraction' and 'relativity of simultaneity' effects arise from misuse of the Lorentz transformation. Consistency with the path-amplitude formulation of quantum mechanics requires the specific covariant Lagrangian recently proposed by Zakharov, Zinchuk and Pervushin.

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The formulae giving the relativistic energy and momentum of a uniformly moving object were derived first, by Planck, in 1906 [1], on the basis of Hamilton's Principle and the Lagrange Equations, from a non-covariant relativistic Lagrangian. Planck's derivation is recalled, and its relation to Einstein's famous formula '$E = mc^2$' discussed, in Appendix A below. Recently a similar calculation was performed by Zakharov, Zinchuk and Pervushin (ZZP) [2]. In this case, not only are relativistic energy and momentum and world-line equations derived, but also, from the Lagrange Equation, corresponding to the temporal component of the space-time four vector, a position-independent time dilation (TD) relation. This TD relation demonstrates that the 'relativity of simultaneity' (RS) and 'length contraction' (LC) effects of conventional special relativity theory are spurious. How they arise from insufficient attention being paid to important additive constants in the Lorentz Transformation (LT) describing a synchronised clock at an arbitary spatial position is explained in Appendix B below.

The present paper repeats the calculations of ZZP, giving particular attention to the operational meaning of the derived equations, in particular, that of the constants of integration that occur in the derivations of the worldline and TD relations. The covariant action of a freely moving physical object of Newtonian mass m may be written as[a]:

$$S_{\rm cov} \equiv \int d\tau L_{\rm cov} \equiv -\frac{m}{2}\int d\tau (V^2 - c^2) \qquad (1)$$

[a]The Lagrangian $L_{\rm cov}$ in (1) differs from that given by Goldstein [3] by an additive constant $mc^2/2$. Also Goldstein employs a space-like metric for 4-vector products in place of the time-like metric assumed in Eq. (1).

where τ is the proper time of the object and its four-vector velocity, V, is defined as

$$V = (V_0; \vec{V}) \equiv \frac{dX}{d\tau} = (c\frac{dt}{d\tau}; \frac{d\vec{X}}{d\tau}) = (c\gamma_v; \vec{v}\gamma_v) \tag{2}$$

where

$$X = (X_0; \vec{X}) \equiv (ct; X_1, X_2, X_3) \equiv (ct; x, y, z),$$

$$\gamma_v \equiv \frac{1}{\sqrt{1-\beta_v^2}}, \quad \beta_v \equiv |\vec{\beta}_v|, \quad \vec{\beta}_v \equiv \frac{\vec{v}}{c}$$

and c is the speed of light in free space. The arbitary constant that may be added to the Lagrangian, without affecting the Lagrange Equations, is chosen in (1) so that, in the non-relativistic (NR) limit where c tends to infinity, $L = T \to T_{\rm NR} = mv^2/2$.

In virtue of the identity $\gamma_v^2 - \gamma_v^2\beta_v^2 \equiv 1$, which implies that $V^2 = V_0^2 - \vec{V}^2 = c^2$, $L_{\rm cov}$ in (1) actually vanishes. However the Lagrange Equations:

$$\frac{d}{d\tau}\left(\frac{\partial L_{\rm cov}}{\partial V_\mu}\right) - \frac{\partial L_{\rm cov}}{\partial X_\mu} = 0 \tag{3}$$

corresponding to a stationary value of the action, are determined only by the functional dependence of $L_{\rm cov}$ on V_μ and X_μ, not on its value. Since $\partial L_{\rm cov}/\partial X_\mu = 0$, these equations are:

$$\frac{dP_\mu}{d\tau} = 0, \quad P_0 \equiv -\frac{\partial L_{\rm cov}}{\partial V_0}, \quad P_i \equiv \frac{\partial L_{\rm cov}}{\partial V_i} \tag{4}$$

where the indices μ (i) denote components of four-(three-) vectors. The definition of $L_{\rm cov}$ in (1) gives for the energy-momentum four-vector $P = (P_0; \vec{P})$:

$$P_0 = mV_0 = mc\gamma_v = mc\frac{dt}{d\tau}, \quad \vec{P} = m\vec{V} = m\gamma_v\vec{v} = m\gamma_v\frac{d\vec{X}}{dt}. \tag{5}$$

Because, from the first equation in (4), P_μ is time-independent, the first-order differential equations in (5) may be integrated to give [2]:

$$t - t_I = \frac{P_0}{mc}(\tau - \tau_I), \quad X_i - X_{iI} = \frac{P_i}{\gamma_v m}(t - t_I) \tag{6}$$

The integration constants t_I and τ_I describe, respectively, the synchronisation of a clock, C, recording time t, at rest in the frame S where the object has velocity $\vec{v}$, and a clock C', recording time τ, at rest in the proper frame, S', of the object. The integration constants X_{iI} depend on the choice of the origin of spatial coordinates in the frame S. Since (5) gives $P_0/mc = \gamma_v$ and $P_i/\gamma_v m = v_i$, the space-time geometry of the uniformly moving object in the inertial frame S is completely specified by the equations:

$$t - t_I = \gamma_v(\tau - \tau_I), \quad X_i - X_{iI} = v_i(t - t_I). \tag{7}$$

The first, time, transformation equation describing the time dilation (TD) effect *contains no spatial coordinates*. The TD effect is therefore a universal one for a pair of clocks at any positions in the frames S and S'. The other equations describing the motion of the object (its worldline) in the frame S are the same as in Galilean relativity. Without loss of generality (because of the isotropy of space) $\vec{X}$ may be chosen parallel to $\vec{v}$, and t_I and τ_I may be chosen so that the transformation equations become:

$$t(\mathrm{C}) = \gamma_v \tau = \gamma_v t(\mathrm{C}'), \quad x(\mathrm{C}') - x(\mathrm{C}')_I = vt(\mathrm{C}). \tag{8}$$

Now the uniformly moving object *is* the clock C', and when $x(\mathrm{C}') = x(\mathrm{C}')_I$ the clocks are synchronised so that $t(\mathrm{C}') = t(\mathrm{C}) = 0$. Since the origin of spatial coordinates in the frame S' may be chosen in an arbitary manner, it is always possible to set $x'(\mathrm{C}') = x(\mathrm{C}')_I = d'$ so that the transformation equations become:

$$x'(\mathrm{C}') = d', \quad x(\mathrm{C}') = vt(\mathrm{C}) + d', \quad t(\mathrm{C}) = \gamma_v t(\mathrm{C}'). \tag{9}$$

If there is another clock, $\tilde{\mathrm{C}}'$, at rest in S' at $x'(\tilde{\mathrm{C}}') = \tilde{d}'$, then, using the same spatial coordinates in S and S' as in (9), the transformation equations are:

$$x'(\tilde{\mathrm{C}}') = \tilde{d}', \quad x(\tilde{\mathrm{C}}') = v_i t(\mathrm{C}) + \tilde{d}', \quad t(\mathrm{C}) = \gamma_v t(\tilde{\mathrm{C}}'). \tag{10}$$

Similiarly to C', $\tilde{\mathrm{C}}'$ is synchronised with C so that when $x(\tilde{\mathrm{C}}') = \tilde{d}'$ then $t(\tilde{\mathrm{C}}') = t(\mathrm{C}) = 0$. The purely mechanical (or electronic) operation of setting the clocks C', $\tilde{\mathrm{C}}'$ and C to a common epoch at some instant can always be performed. The TD relation predicts only the relative *rate* of clocks at rest in S and S', not their settings [4]. It follows from the TD relations in (9) and (10) that $\gamma_v t(\mathrm{C}') = t(\mathrm{C}) = \gamma_v t(\tilde{\mathrm{C}}')$ or that $t(\mathrm{C}') = t(\tilde{\mathrm{C}}')$ so that the clocks C' and $\tilde{\mathrm{C}}'$, at different postions in S', are synchronised (i.e. record the same epoch) for an arbitary value of the epoch $t(\mathrm{C})$ —there is no 'relativity of simultaneity' (RS) effect. Subtracting the first equation in (9) from the first in (10) and the second in (9) from the second in (10) gives:

$$x'(\tilde{\mathrm{C}}') - x(\mathrm{C}') \equiv L' = \tilde{d}' - d' = x(\tilde{\mathrm{C}}') - x(\mathrm{C}') \equiv L. \tag{11}$$

The spatial separation of the clocks is therefore the same in S and S' at all times —there is no 'length contraction' (LC) effect. How the spurious RS and correlated LC effects arise in conventional special relativity theory is explained in Appendix B.

The equations (9) and (10) provide a complete description of the relativistic space-time geometry of the clocks C' and $\tilde{\mathrm{C}}'$ in the inertial frames S and S'. To see the equivalence of these formulae to the more familiar Lorentz transformation (LT) equations it may be noted that, considering the clock C':

$$t(\mathrm{C}) - \frac{v(x(\mathrm{C}') - d')}{c^2} = t(\mathrm{C})(1 - \frac{v^2}{c^2}) = \frac{t(\mathrm{C})}{\gamma_v^2} = \frac{t(\mathrm{C}')}{\gamma_v} \tag{12}$$

or

$$t(\mathrm{C}') = \gamma_v \left[t(\mathrm{C}) - \frac{v(x(\mathrm{C}') - d')}{c^2} \right] \tag{13}$$

which is the time LT with a particular choice of spatial coordinates and clock synchronisation constants. The corresponding space transformation follows immediately from the first two (worldline) equations in (9):

$$x'(\mathrm{C}') - d' = \gamma_v[x(\mathrm{C}') - d' - v_i t(\mathrm{C})] = 0. \tag{14}$$

This equation also holds true if γ_v is replaced by any other (non-infinite) constant showing that all relevant physical information is already to be found in the worldline equations.

The description of the space-time geometry of a uniformly moving object is now complete. Also the components of the energy-momentum four-vector in (5) have been derived from the Lagrange Equations (4). The transformation equations of the components of $\vec{P}$ are now derived by first considering the transformation law of the TD factor γ. For this a particular configuration of uniformly moving clocks is considered that actually corresponds to the one realised in the Hafele-Keating experiment performed in 1971 [5] where the time intervals recorded by airborne clocks circumnavigating the Earth were compared with those registered by clocks on the surface of the Earth. Three clocks, C, C$'$ and C$''$ are considered. The first two are at rest in the inertial frames S and S' considered previously, where C$'$ moves with speed v along the positive x-axis in S. The clock C$''$ moves with speed u' in the frame S' at an arbitary angle θ' relative to the x'-axis so that $u'_{x'} = u' \cos\theta'$[b]. Without loss of generality it may be considered to move in the x'-y' plane.

The calculation is based on a generalisation of the inverse interval transformation equation:

$$\Delta t(\mathrm{C}) = \gamma_v \left[\Delta t(\mathrm{C}') + \frac{v\Delta x'(\mathrm{C}')}{c^2}\right] \tag{15}$$

where $\Delta t(\mathrm{C}) \equiv t_2(\mathrm{C}) - t_1(\mathrm{C})$ etc, that may be derived by eliminating $\Delta x(\mathrm{C}')$ from the interval LT equations that may be derived from (13) and (14). Since $\Delta x'(\mathrm{C}') = 0$, (15) is in fact equivalent to the TD relation in (9). Suppose now that the intervals $\Delta t(\mathrm{C})$, $\Delta t(\mathrm{C}')$ and $\Delta x'$ in (15) correspond to pairs of points on the world line of C$''$. In this case the following relations hold:

$$\Delta x'(\mathrm{C}'') = u'_{x'}\Delta t(\mathrm{C}'), \tag{16}$$
$$\Delta t(\mathrm{C}') = \gamma_{u'}\Delta t(\mathrm{C}'') \equiv \gamma'(\mathrm{C}'')\Delta t(\mathrm{C}''), \tag{17}$$
$$\Delta t(\mathrm{C}) = \gamma_v \Delta t(\mathrm{C}') = \gamma(\mathrm{C}'')\Delta t(\mathrm{C}'') \equiv \gamma_u \Delta t(\mathrm{C}''). \tag{18}$$

Combining (15)-(18) leads to the following transformation law for the TD factor for the clock C$''$ between the frames S' and S:

$$\gamma(\mathrm{C}'') = \gamma(\mathrm{C}')[\gamma'(\mathrm{C}'') + \beta(\mathrm{C}')\beta'_{x'}(\mathrm{C}'')\gamma'(\mathrm{C}'')] \equiv \gamma_u = \gamma_v\gamma_{u'}[1 + \beta_v\beta_{u'_{x'}}] \tag{19}$$

where

$$\gamma(\mathrm{C}') \equiv \gamma_v, \quad \beta(\mathrm{C}') \equiv \beta_v, \quad \beta'_{x'}(\mathrm{C}'') \equiv \beta_{u'_{x'}}.$$

A transparent notation is introduced here for the TD factors γ and scaled velocities β in which the observed moving clock, and the frame from which the TD effect is viewed, are explicitly specified.

Since transverse spatial intervals are invariant under transformation between the frames S and S', the y component of the four-vector velocity of C$''$ is invariant:

$$U_y = \frac{dy}{d\tau} = \frac{dy'}{d\tau} = U'_{y'} \tag{20}$$

it follows from (20) that

$$\gamma_u u_y = \gamma_{u'} u'_{y'} \tag{21}$$

[b]In the Hafele-Keating experiment the clock C$''$ was located in the aircraft, C$'$ at a fixed position on the surface of the Earth and C was a hypothetical clock recording 'coordinate time' in a non-rotating frame comoving with the centroid of the Earth.

which, when combined with (19), gives the transformation law:

$$u_y == \frac{u'_{y'}}{\gamma_v(1+\beta_v\beta_{u'_{x'}})}. \tag{22}$$

The definition of γ_v and (19) and (22) give:

$$\begin{aligned}\beta_{u_x} &= \sqrt{\beta_u^2-\beta_{u_y}^2} = \left[1-\frac{1}{\gamma_u^2}-\left(\frac{\beta_{u'_{y'}}}{\gamma_v(1+\beta_v\beta_{u'_{x'}})}\right)^2\right]^{\frac{1}{2}} \\ &= \left[1-\frac{(1-\beta_v^2)(1-\beta_{u'_{x'}}^2)}{(1+\beta_v\beta_{u'_{x'}})^2}\right]^{\frac{1}{2}} = \frac{\beta_v+\beta_{u'_{x'}}}{1+\beta_v\beta_{u'_{x'}}}.\end{aligned} \tag{23}$$

Multiplying both sides of (19) by c and using the definition (2) of a four-vector velocity gives the transformation law of the temporal component of U:

$$U_0 = \gamma_v(U'_{0'}+\beta_v U'_{x'}). \tag{24}$$

The transformation law of the longitudinal component of U is given by (19) and (23) as:

$$U'_{x'}+\beta_v U'_{0'} = c\gamma_{u'}(\beta_{u'_{x'}}+\beta_v) = c\gamma_{u'}(1+\beta_v\beta_{u'_{x'}})\beta_{u_x} = \frac{\gamma_u\beta_{u_x}}{\gamma_v} \tag{25}$$

or, transposing,

$$\gamma_u\beta_{u_x} = U_x = \gamma_v(U'_{x'}+\beta_v U'_{0'}). \tag{26}$$

Note that the formulae (23), (19) or (24), and (26) are algebraically equivalent, i.e. on posing any one of them, the other two may be derived by purely algebraic manipulation.

The transformation laws of the energy-momentum four vector follow from the definitions in (5) and the transformation laws (20), (24) and (26) of the components of the velocity four-vector:

$$P_0 = \gamma_v(P'_{0'}+\beta_v P'_{x'}), \quad P_x = \gamma_v(P'_{x'}+\beta_v P'_{0'}), \quad P_y = P_{y'}. \tag{27}$$

The identity $\gamma_u^2 - \gamma_u^2\beta_u^2 \equiv 1$ and the relations in (5) between the components of the energy-momentum four-vector and the velocity four-vector give

$$m^2c^4 = c^2P_0^2 - c^2\vec{P}^2 \tag{28}$$

or, setting $E \equiv cP_0 = \gamma_u mc^2$,

$$E(m,P) = \sqrt{m^2c^4+P^2c^2} \tag{29}$$

where $P \equiv |\vec{P}|$. If $P \ll mc^2$, (29) gives

$$E(m,P) \simeq mc^2 + \frac{P^2}{2m} \tag{30}$$

so that $E(m,P=0) = mc^2$, which is the operationally correct statement of Einstein's famous formula, in which the mass m is Lorentz scalar independent of the velocity of the object [6].

It is important to stress that there is no conflict between the above assertion of the absence of RS and LC and the results of existing experimental tests of special relativity. This important question is addressed in Section 9 of Ref. [7]. Different experimental tests of special relativity may be categorized as follows:

(i) Isotropy and source independence of the speed of light.

(ii) Measurements of the relativistic Doppler effect.

(iii) Tests of relativistic kinematics.

(iv) Tests of time dilation.

(v) Tests in lepton $g-2$ experiments.

(vi) Tests of length contraction in particle production models.

None of these has provided any evidence for the existence of the RS and LC effects. The conventional text book relativistic analysis of the Michelson-Morley experiment (MME) is cited in Ref. [8] as evidence for the LC effect. However, as discussed in Section 8 of Ref. [7], the putative contraction of the longitudinal arm in the MME is not only the result of a calculation different that by which LC is derived from misuse of the LT but also implies lack of space-time contiguity of emission events and their sources in different inertial frames. The correct relativistic analysis of the MME [7] shows that there is no contraction of the longitudinal arm. Many experiments have verified the TD effect both for macroscopic clocks [5] and for the observed lifetimes of elementary particles in motion [9, 10]. The kinematical transformation formulae (24) and (26) are verified in many different experiments in the categories (ii) and (iii) above. The most recent of these experiments are cited in Ref. [7], earlier ones in Ref. [8]. In agreement with Feinberg [12] it is concluded in Ref. [7] that no convincing evidence for LC is provided by the tests in category (vi).

Although at this time of writing no experimental test of RS exists, since this is an $\mathrm{O}(\beta)$ (not an $\mathrm{O}(\beta^2)$) effect, it can readily be tested using modern high precision clocks. Such tests, using artificial satellites of the Earth, have been proposed. [7, 11] by the present author.

In Ref. [2], ZZP considered the covariant action[c]:

$$S_{\rm cov}^{\rm ZZP} \equiv \int d\tau L_{\rm cov}^{\rm ZZP} \equiv -\frac{m}{2}\int d\tau e(\tau)\left[\left(\frac{dX_\mu}{e(\tau)d\tau}\right)^2+1\right] \tag{31}$$

where units with $c=1$ are used. Requiring that the action is stationary with respect to variation of $e(\tau)$ gives

$$[e(\tau)d\tau]^2 = dX_\mu^2 = (d\tau)^2 \tag{32}$$

so that $e(\tau)=1$. Restoring the units and definitions of the action in Eq. (1) then gives:

$$\begin{aligned} S_{\rm cov}^{\rm ZZP} &= -\frac{m}{2}\int d\tau[V^2+c^2] = -mc^2\int d\tau = -mc^2\int\frac{dt}{\gamma_v} \\ &= -mc^2\int dt\sqrt{1-\beta_v^2} \equiv S_{\rm rel}^{\rm Plnk}. \end{aligned} \tag{33}$$

The action $S_{\rm cov}^{\rm ZZP}$ is then actually identical (up to an arbitary additive constant) to the one $S_{\rm rel}^{\rm Plnk}$ introduced by Planck [1] which contains the non-covariant relativistic Lagrangian, L, in Eq. (A1) of Appendix A.

[c]Notice that this Lagrangian differs from that in (1) by an additive term $-mc^2$ and from that given by Goldstein [3] by an additive term $-mc^2/2$. In Ref. [2] the action of (31) was associated with Hilbert's General Relativity paper [13] and called $S_{\rm SR:1915}$ where 1915 is the publication year of Hilbert's paper. However careful examination of the latter fails to reveal any formula resembling Eq. (31). In Ref. [2] a paper [14], cited in connection with Eq.(31), contains to citation or mention of Hilbert's paper.

Because an arbitary constant may be added to a Lagrangian without changing the Lagrange equations, identical predictions are obtained from $S_{\rm cov}$ in Eq. (1) and $S_{\rm cov}^{\rm ZZP}$ in Eq. (40). However, in Feynman's path amplitude formulation of quantum mechanics, the covariant space-time propagator $K(X_\mu)$ of a free particle has the asymptotic form [15, 16]:

$$K(X_\mu) \simeq \exp\left[-i\frac{(Et-px)}{\hbar}\right] = \exp\left[-i\frac{mc^2\tau}{\hbar}\right] = \exp\left[i\frac{S_{\rm cov}^{\rm ZZP}}{\hbar}\right] \qquad (34)$$

since, from (33), $-mc^2\tau = S_{\rm cov}^{\rm ZZP}$. In the Feynman propagator, therefore, the redundancy in the definition of the Lagrangian and the corresponding action, allowed in classical mechanics, is removed.

It is also interesting to remark that, as first pointed out by Dirac [17, 18], it is the behaviour of $K(X_\mu)$ in the limit $\hbar \rightarrow 0$ that provides the fundamental quantum mechanical basis for Hamilton's Principle —the condition that the action should be stationary for variation of space-time trajectories around the classical path. From this point-of-view the derivations of Planck, ZZP and the present paper of space-time geometry or relativistic kinematics can all be considered as necessary consequences of quantum mechanics in its path-amplitude formulation [19, 20, 16].

Appendix A

Planck's derivation of the formulae of relativistic kinematics for a uniformly moving object [1] is recalled here using the notation of the present paper. A non-covariant relativistic Lagrangian, L, for a free particle was defined as:

$$L \equiv -mc^2\sqrt{1-\frac{v^2}{c^2}} + \quad \text{const.} \tag{A.1}$$

where

$$v^2 = \left(\frac{d\vec{X}}{dt}\right)^2 \equiv (\dot{\vec{X}})^2. \tag{A.2}$$

The relativistic three-momentum is then given by the Lagrange Equations as

$$P_i \equiv \frac{\partial L}{\partial \dot{X}_i} = \frac{m\dot{X}_i}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{A.3}$$

If H denotes the Hamiltonian of the object then

$$\begin{aligned} H(m,v) &\equiv \vec{P}\cdot\dot{\vec{X}} - L = m\left[\frac{\dot{\vec{X}}^2}{\sqrt{1-\frac{v^2}{c^2}}} + c^2\sqrt{1-\frac{v^2}{c^2}}\right] + \quad \text{const.} \\ &= \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left[\frac{v^2}{c^2} + 1 - \frac{v^2}{c^2}\right] + \quad \text{const.} \\ &= \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + \quad \text{const.}\ . \end{aligned} \tag{A.4}$$

Combining (A.3) and (A.4) gives:

$$H(m,P) = mc^2\sqrt{1+\frac{P^2}{m^2c^2}} + \quad \text{const.}\ . \tag{A.5}$$

Einstein had previously derived [22] the formula for the relativistic kinetic energy, T, of an object:

$$T(m,v) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2. \tag{A.6}$$

Identifying, as in Newtonian classical mechanics, the Hamiltonian with the energy, E, of the object and setting the arbitary constant in (A.5) to zero gives then:

$$E(m,v) \equiv H(m,v) = E(m,v) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{A.7}$$

so that

$$E(m, v=0) = mc^2 \tag{A.8}$$

the fundamental equation that Einstein had so much difficulty to derive in a rigorous manner [6, 21].

Planck did not mention Eq. (A8), deriving only (A.3), (A.4) and (A.5). It still follows however on setting to zero the arbitary constant in (A.5) that

$$E(m, P = 0) \equiv H(m, P = 0) = mc^2. \tag{A.9}$$

Defining:

$$\begin{aligned} T(m, v) &\equiv E(m, v) - E(m, v = 0) \\ &= E(m, v) - E(m, P = 0) \\ &= E(m, v) - mc^2 \end{aligned} \tag{A.10}$$

recovers Einstein's formula (A.6).

Planck therefore derived the formula for the relativistic momentum of an object from Hamilton's Principle, as done by ZZP and in the present paper. Because Planck employed a non-covariant Lagrangian it was not possible to obtain the relativistic energy directly from the Lagrange Equations, as in (4) and (5) above. Instead Planck constructed the relativistic Hamiltonian from the postulated Lagrangian and the derived relativistic momentum. Identifying this Hamiltonian with the relativistic energy then gives the formula (A.5) for the latter in terms of m and P. Einstein's famous '$E = mc^2$' formula relating the energy of an object to its rest mass then follows from (A5) on setting the arbitary additive constant to zero. It is clear in this derivation that the mass m is a Lorentz scalar quantity [6]. Finally, it may be remarked that, as described above, consistency with quantum mechanics *requires* the additive constants in (A.1), (A.4) and (A.5) to be zero.

Appendix B

The LT describing the clock C′, as observed from the frame S, (13) and (14) above, may be rewritten as:

$$t'(d') = \gamma_v \left[t(\mathrm{C}) - \frac{v(x(d')-d')}{c^2} \right], \tag{B.1}$$

$$x'(d') - d' = \gamma_v [x(d') - d' - vt(\mathrm{C})] = 0 \tag{B.2}$$

where the clock C′ has been labelled with its x' coordinate. Independently of the value of d', the clock C′ is synchronised with the clock C so that when $x(d') = d'$, $t'(d') = t(\mathrm{C}) = 0$. The conventional space-time LT is given by setting $d' = 0$ in (B.1) and (B.2):

$$t'(0) = \gamma_v \left[t(\mathrm{C}) - \frac{vx(0)}{c^2} \right], \tag{B.3}$$

$$x'(0) = \gamma_v [x(0) - vt(\mathrm{C})] = 0. \tag{B.4}$$

The spurious RS and LC effects arise when it is incorrectly assumed that (B.3) and (B.4) *also correctly describe a synchronised clock at an arbitary position $x' = d'$.* i.e. that

$$t'(d') = \gamma_v [t(\mathrm{C}) - \frac{vx(d')}{c^2}] \tag{B.5}$$

$$x'(d') = \gamma_v [x(d') - vt(\mathrm{C})] = d' \neq 0. \tag{B.6}$$

Subtracting (B4) from (B6) gives:

$$x'(d') - x'(0) = d' = \gamma_v [x(d') - x(0)] \equiv \gamma_v d \tag{B.7}$$

which is the 'LC' effect, while subtracting (B.3) from (B.5) gives

$$t'(d') - t'(0) = -\gamma_v \frac{v[x(d') - x(0)]}{c^2} = -\frac{vd'}{c^2} \tag{B.8}$$

which is a 'RS' effect since the common time $t(\mathrm{C})$ in S, in Eqs. (B.3)-(B.6), gives, according to (B.8), $t'(d') \neq t'(0)$.

Both (B.1), (B.2) and (B.3), (B.4) do correctly describe a clock that is synchronised with C such that $t'(d') = t(\mathrm{C}) = 0$ when $x(d') = d'$ (for (B.3) and (B.4), $d' = 0$). To see the relation between the correct equations describing a synchronised clock at $x' = d'$, (B.1) and (B.2), and the incorrect equations (B.5) and (B.6), the former pair of equations are written as:

$$t'(d') = \gamma_v \left[t(\mathrm{C}) - \frac{vx(d')}{c^2} \right] + T, \tag{B.9}$$

$$x'(d') = \gamma_v [x(d') - vt(\mathrm{C})] + X \tag{B.10}$$

where

$$T = \frac{v\gamma_v d'{}'}{c^2}, \tag{B.11}$$

$$X = -d'(\gamma_v - 1). \tag{B.12}$$

Thus the correct equations describing synchronised clocks in S and S', (B.1) and (B.2) differ from the 'standard LT equations' (B.5) and (B.6) by, velocity-dependent, additive, constants on the right sides of the latter equations. The necessity to include such constants, in order to correctly describe synchronised clocks at different spatial positions, was clearly stated by Einstein himself in the original 1905 special relativity paper just after the derivation of a LT equivalent to (B.3) and (B.4) above (i.e. one with $d' = 0$) [22]:

"If no assumption whatever be made as to the initial position of the moving system and as to the zero point of τ an additive constant is to be placed on the right side of these equations"[d]

To the present writer's best knowledge, this was never done either by Einstein himself, or any other author, before the work presented in Ref. [7]. The analysis of the present paper, which does not make use of the space-time LT, and in which the important additive constants appear in the guise of constants of integration in worldline and TD relations, such as (9) and (10), demonstrates directly the spurious nature of the 'RS' and 'LC' effects of conventional special relativity.

[d]In the notation of the present paper, $\tau = t' = t(\mathrm{C}')$.

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