

Proper consideration of the Minkowski space-time plot demonstrates the spurious nature of the ‘relativity of simultaneity and ‘length contraction’ effects of conventional special relativity theory

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Abstract

It is demonstrated that the relativity of simultaneity and length contraction effects of special relativity are artifacts of the use of different coordinate systems to specify the spatial positions of separated physical objects in their common rest frame.

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In this letter, observation, in different inertial frames, of events lying on the worldlines of physical objects are considered with particular attention to the exact operational definitions of the symbols representing times (clock epochs) and spatial coordinates in the Lorentz transformation equations. Two inertial frames S, S' are considered where S' moves with speed v along the the positive x -axis of S and the x , x' axes are parallel [1]. Only objects, at rest in S', lying on the x' axis, are considered. Following Taylor and Wheeler [2] it is assumed that both inertial frames are equipped with a dense network of synchronised clocks all registering, at any instant, the same epoch: t in S and t' in S', so that the time of an event at an arbitrary position in either frame may be identified with that registered by a spatially-adjacent clock. Events in S are denoted as (x,t) , those in S' by (x',t') .

The experiment to be considered is one in which an observer at rest in the frame S compares the frame time, t , registered by the clocks at rest in S with that, t' , registered by clocks at rest in S', for events that lie on the worldlines of specific physical objects. The free parameters defining the initial conditions of the experiment are the value of v , the initial difference, t_0 , between the frame times t and t' and the initial positions of the objects in the frame S. The quantities to be calculated are (i) the relation between t' and t for arbitrary values of t and (ii) the separation of the two objects in the frame S' when their separation in the frame S is defined. The formulas giving both of these relations are found to be invariant with respect to the choice of the origins of coordinates in both S and S', i.e. to respect translational invariance.

The worldlines of an arbitrary object, labelled i , at rest in S', are, in the frames S' and S respectively:

$$x'_i(t'_i) = x'_i(0), \quad x_i(t_i) = v(t_i - t_0) + x_i(t_0). \quad (1)$$

The values of the constants $x'_i(0)$, $x_i(t_0)$ depend on the choice of coordinate origins in S and S', and that of t_0 on the relative synchronisation of the clock arrays in S and S'. The

time Lorentz transformation for events on the worldline of the object i is

$$t'_i = \gamma \left[t_i - t_0 + \frac{v(x_i(t_i) - x_i(t_0))}{c^2} \right] \quad (2)$$

where $\gamma \equiv 1/\sqrt{1 - \beta^2}$, $\beta \equiv v/c$ and c is the speed of light in free space. The standard time transformation equation [1] $t'_i = \gamma(t_i - vx_i(t_i)/c^2)$ is recovered on setting $t_0 = 0$, $x_i(t_0) = 0$, i.e. for a particular choice of the coordinate origin in S and of the relative synchronisation of the frame times t and t' . As discussed below, (2) is obtained from the standard transformation equation by making the coordinate replacement: $x_i(t_i) \rightarrow x_i(t_i) - x_i(t_0)$ i.e. by making a different choice for the position of the origin of coordinates in S. Using the worldline of i in S (the second equation in (1)) to eliminate the spatial coordinate $x_i(t_i)$ from (2) and rearranging, using the definition of γ , gives the time dilation (TD) relation, that is the result (i) mentioned above:

$$t_i - t_0 = \gamma t'_i. \quad (3)$$

Notice that, unlike the time transformation equation (2), the TD relation contains no spatial coordinates, and so is independent of the value of the parameter $x_i(t_0)$ that depends on the choice of the origin of coordinates in S. The time dilation relation is therefore a translational invariant. The physical meaning of the parameter t_0 can be read off from (3). When $t = t_0$, then $t'_i = 0$ which implies that the settings of the S-frame clocks are in advance of the S'-frame ones by t_0 units when $t'_i = 0$.

Combining (3) with the worldline equation of object i in (1) gives:

$$x_i(t_i) - x_i(t_0) = \beta \gamma c t'_i. \quad (4)$$

Squaring both sides of (3) and (4), subtracting and making use of the identity: $\gamma^2 - \gamma^2 \beta^2 \equiv 1$ gives:

$$c^2(t'_i)^2 = c^2(t_i - t_0)^2 - [x_i(t_i) - x_i(t_0)]^2. \quad (5)$$

This is the familiar hyperbolic curve on the ct versus x Minkowski plot [3] relating an event with fixed time t'_i in S' to corresponding events $(x_i(t_i), t_i)$ in the frame S, for arbitrary values of the velocity parameter v . We are now in a position to investigate the simultaneity properties in S and S' of events on the world lines of two spatially-separated objects labelled 1 and 2. Simultaneous events in the frame S' on the worldlines of 1 and 2 such that $t'_1 = t'_2 = t'$ are considered and (5) and the worldline equations in S are used to calculate the corresponding times in the frame S. For such events (5) gives:

$$c^2(t')^2 = c^2(t_1 - t_0)^2 - [x_1(t_1) - x_1(t_0)]^2 = c^2(t_2 - t_0)^2 - [x_2(t_2) - x_2(t_0)]^2. \quad (6)$$

The hyperbolas $H_1(t', x_1, t_1)$, $H_2(t', x_2, t_2)$ given by (6) for the objects 1 and 2 are plotted in Fig.1 for the case $t_0 = 0$ together with the world lines, in the frames S, of 1 and 2 for $\beta = 0.0, 0.5$ and 0.75 . The space time events in S corresponding to the fixed epoch, t' , in S' are given by the intersections of the worldlines with H_1 and H_2 . Inspection of Fig.1 shows that the world lines of 1 and 2 with the same value of β intersect the hyperbolas at the same value of t : $t_1 = t_2 = t$. Events which are simultaneous in S' are therefore also simultaneous in S —there is no 'relativity of simultaneity' (RS) effect.

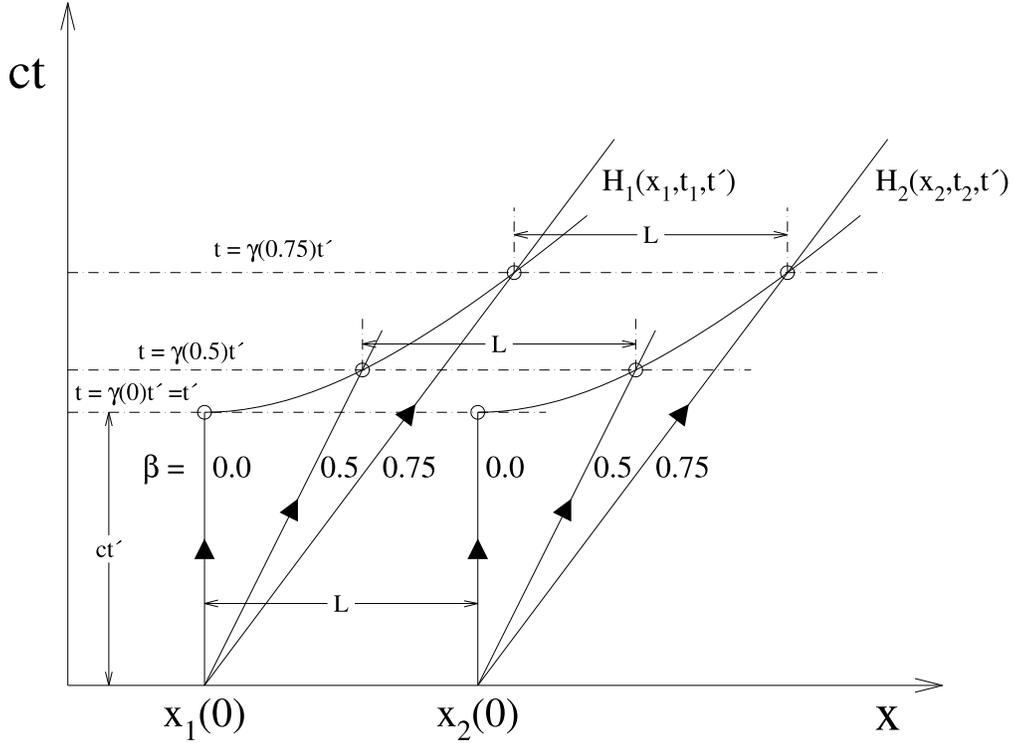


Figure 1: *Minkowski space time plot of events (x, ct) in the frame S . The hyperbolae H_1 and H_2 are the loci of events on the worldlines of objects 1 and 2 for a fixed value of t' and arbitrary values of $\beta = v/c$. Also shown (arrowed lines) are the world lines of the objects for $\beta = 0, 0.5$ and 0.75 . The absence of any *RS* effect is manifest by the equality of the values of t given by the intersections of the worldlines with the hyperbolae for the different values of β . The hyperbolae and worldlines of object 2 are obtained from those of object 1 by the transformations: $x_2 = x_1 + L$, $t_2 = t_1$ that manifest the translational invariance of the space-time geometric effects shown in the figure.*

With a suitable choice of the origin of coordinates in S^a —which leaves invariant all physical predictions— the worldline of the object i in the frame S' can be written $x'_i(t'_i) = x_i(t_0)$, giving, for the objects 1 and 2 the worldlines:

$$x'_1(t'_1) = x_1(t_0), \quad x_1(t_1) = v(t_1 - t_0) + x_1(t_0) \quad (7)$$

$$x'_2(t'_2) = x_2(t_0), \quad x_2(t_2) = v(t_2 - t_0) + x_2(t_0). \quad (8)$$

It follows from these equations that:

$$L' \equiv x'_2(t'_2) - x'_1(t'_1) = x_2(t_0) - x_1(t_0) = x_2(t) - x_1(t) \equiv L. \quad (9)$$

for all values of t'_1 and t'_2 . The spatial separation of 1 and 2 is therefore the the same in the frames S' and S —there is no ‘length contraction’ (LC) effect. This is the result (ii) mentioned above. Note that the relation (9) is invariant with respect to the choice of coordinate systems in the frames S and S' , i.e. under the transformations $x \rightarrow x + X$, $x' \rightarrow x' + X'$ for arbitrary values of X and X' .

A physical illustration of the invariance of the length interval between two physical objects, in different inertial frames, without consideration of specific coordinate systems, or space-time transformation equations, is provided by the thought experiment shown in Fig. 2. Two small objects O1 and O2 lying on the x -axis in the frame S are intially aligned with two ‘marker objects’ M1 and M2, that have fixed positions in this frame. The objects O1 and O2 then undergo simultaneous and equal acceleration programs during which both move a distance d in the frame S , and after which they move with the same constant velocity. Fig. 1a shows the experiment as viewed in the frame S and Fig. 1b the experiment as viewed in the common comoving inertial frame S' of O1 and O2. The acceleration ceases at time $t = t_{acc}$ in S and at time $t' = t'_{acc}$ in S' . The qualitaive features of Fig. 1 are the same for any acceleration program, but for definitness the case of ‘hyperbolic acceleration’ where where the velocity in the frame S is given, as a function of t , by the relation

$$\beta(t) = \frac{at}{\sqrt{c^2 + a^2t^2}} \quad (10)$$

where a is a constant with dimensions of acceleration, is considered. Time integration of the velocity and use of the TD dilation relation for the instantaneous velocity gives:

$$d = \frac{c^2}{a} \left[\sqrt{1 + \left(\frac{at_{acc}}{c} \right)^2} - 1 \right] = \frac{c^2}{a} \left[\cosh \left(\frac{at'_{acc}}{c} \right) - 1 \right]. \quad (11)$$

Choosing parameters and units such that $a = c = 1$ and $t_{acc} = \sqrt{3} = 1.732\dots$ gives $d = 1$ and $t'_{acc} = 1.317\dots$. In Fig. 2 the invariant interval between O1 and O2 or M1 and M2 is chosen to take the value $L = 4$. The Lorentz invariant character of the separation of the pairs of objects —a necessary consequence of the identical and simultaneous accelerations they undergo, and independent of both the acceleration program and of the form of the space-time transformation equations— is evident from inspection of Fig. 2.

The erroneous derivations of the LC and RS effects to be found in textbooks are now discussed. The typical derivation of LC as given by Landau and Lifshitz [4] is

^aWith this choice the coordinate origins in S and S' are aligned when $t' = 0$, as is also the case for the standard transformation equation.

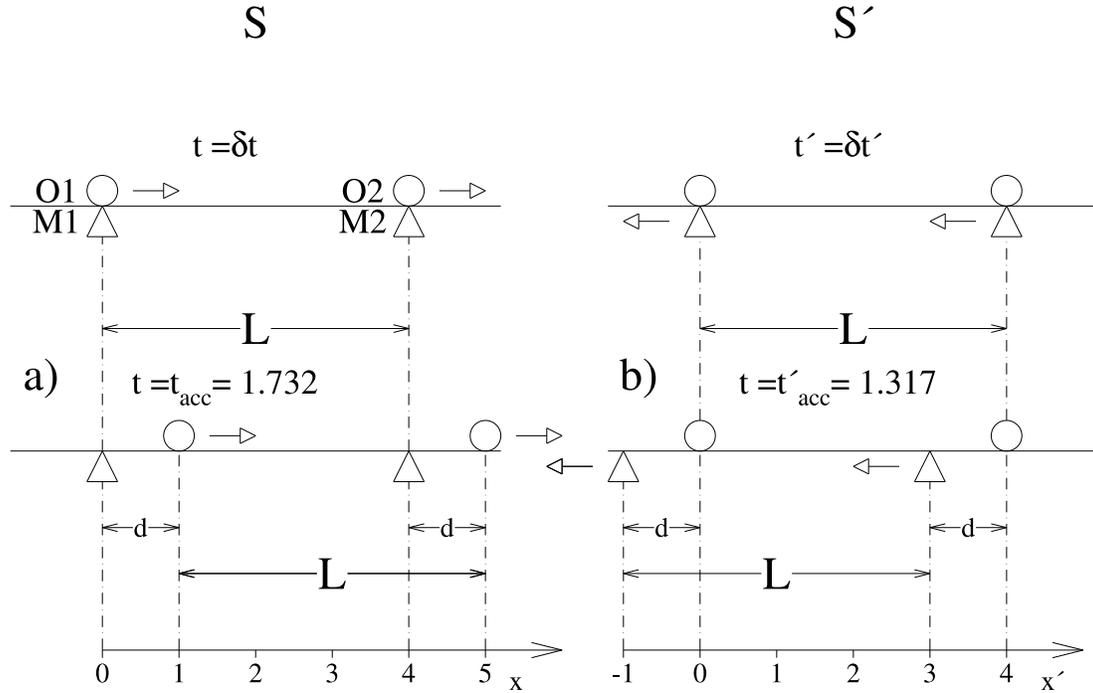


Figure 2: A simple thought experiment demonstrating the Lorentz invariance of the spatial separations of the pairs of objects $O1$ and $O2$ at rest in the frame S' and $M1$ and $M2$ at rest in the frame S . The objects $O1$ and $O2$ undergo an identical and simultaneous acceleration program in the frame S as indicated by the short arrows; a), as viewed from the frame S , b), as viewed from the common instantaneous comoving frame of $O1$ and $O2$. The upper figures show configurations shortly after the start of the acceleration program at times $\delta t \ll t_{acc}$ and $\delta t' \ll t'_{acc}$. The lower figures show configurations at the end of the acceleration programs at times t_{acc} in S and t'_{acc} in S' , when S' is an inertial frame. The manifest invariance of the spatial separations of $O1$ and $O2$ and $M1$ and $M2$ in both frames is a necessary consequence of the assumed equality of the acceleration undergone by $O1$ and $O2$. It is independent both of the acceleration program and of the form of the space-time transformation equations. See text for further discussion.

considered but similar derivations are to be found in all books treating special relativity theory from popular expositions [5] to advanced monographs [6]. The LC effect follows immediately on substituting different arbitrary space time events in the standard space Lorentz transformation equations ^b:

$$x'_i = \gamma(x_i - vt_i), \quad i = 1, 2. \quad (12)$$

On setting $t_1 = t_2 = t$ in order to define a ‘length measurement’ at a fixed instant in the frame S, subtracting the two equations in (12) gives:

$$x'_2 - x'_1 \equiv L' = \gamma(x_2 - x_1) \equiv \gamma L \quad (13)$$

from which it is concluded that the measured length in S is less than the distance between the objects in the frame S' by the factor $1/\gamma$. This is the LC effect. In the calculation, the objects on the worldlines of which the events are located are assumed to be at rest in the frame S'. This means that the coordinates x'_1 and x'_2 are time-independent constants. The worldlines of the corresponding objects in the frame S are then

$$x_1 = vt_1 + x'_1/\gamma, \quad x_2 = vt_2 + x'_2/\gamma \quad (14)$$

from which follows $x_1(t = 0) = x'_1/\gamma$, $x_2(t = 0) = x'_2/\gamma$. The initial conditions in the S-frame worldline equation in Eq. (1) are therefore:

$$t_0 = 0, \quad x_1(t_0) = x'_1/\gamma, \quad x_2(t_0) = x'_2/\gamma. \quad (15)$$

However, as discussed above, the initial conditions of the problem are completely defined by t_0 , $x_1(t_0)$ and $x_2(t_0)$. If the coordinates $x_1(t_0)$ and $x_2(t_0)$ are referred to a common origin in S and $x'_1(0)$ and $x'_2(0)$ to a common origin in S' then Eq. (9) gives:

$$x_2(0) - x_1(0) = L = x'_2(0) - x'_1(0) = (x'_2 - x'_1)/\gamma. \quad (16)$$

The last member of this equation implies that there is a distance δ between the origins of the coordinate systems in S' used to specify x'_2 and x'_1 such that $x'_2 - x'_1 = L' = L + \delta$, which, for consistency with (16), requires that $\delta = (\gamma - 1)L$ and therefore that $L' = \gamma L$. The coordinates x'_2 and x'_1 in Eq. (12) are therefore specified in different coordinate systems, with origins separated by the distance $(\gamma - 1)L$. This is the origin of the LC effect, not a genuine difference of measured length intervals in the two frames. It is a consequence, as will be seen below, of neglect, in the standard Lorentz transformation equations, of important additive constants that must be included in order to correctly describe the initial conditions of the space-time experiment in which the length intervals are measured.

The standard time Lorentz transformations corresponding to the space transformations in (12) are:

$$t'_i = \gamma(t_i - vx_i/c^2), \quad i = 1, 2. \quad (17)$$

Using the worldline equations in (14) to eliminate x_1 and x_2 gives:

$$t'_1 = \frac{t_1}{\gamma} - \frac{vx'_1}{c^2}, \quad t'_2 = \frac{t_2}{\gamma} - \frac{vx'_2}{c^2} \quad (18)$$

^bFor clarity, roman symbols are used for space and time coordinates and length intervals in the standard Lorentz transformation equations, as derived by Einstein [1], where no additive constants specifying initial conditions are included.

to be compared with the time dilation relation (3) with $t_0 = 0$, which is $t_i = \gamma t'_i$. The equations (18) yield the absurd predictions that the relation between the times t' and t on the world lines of the objects 1 and 2 depends on the values of x'_1 and x'_2 respectively, i.e. on the (arbitrary) choice of the origin of spatial coordinates used to specify say x'_1 in the frame S' ! Setting $t_1 = t_2$ and subtracting the first equation in (18) from the second yields the RS prediction:

$$t'_2 - t'_1 = -\frac{v(x'_2 - x'_1)}{c^2} = -\frac{vL'}{c^2} = -\frac{\gamma vL}{c^2} \quad (t_1 = t_2) \quad (19)$$

since $t'_1 \neq t'_2$ when $t_1 = t_2$.

The transformation equations for space-time coordinates in (12) and (17) can be combined to give the transformation equations for space and time intervals that are independent of the choice of coordinate origins in S and S' :

$$\Delta x' = \gamma(\Delta x - v\Delta t), \quad \Delta t' = \gamma(\Delta t - v\Delta x/c^2) \quad (20)$$

where $\Delta x' \equiv x'_2 - x'_1$ etc, which yield the space-like invariant interval relations:

$$(\Delta s)^2 = (\Delta x')^2 - c^2(\Delta t')^2 = (L')^2 - c^2(\Delta t')^2 = (\Delta x)^2 - c^2(\Delta t)^2 > 0. \quad (21)$$

Setting $\Delta x = L$, when $\Delta t = 0$ and $\Delta t' = -\gamma vL/c^2$, from (19), gives, on eliminating $\Delta t'$ from (21):

$$L^2 = (L')^2 - \gamma^2\beta^2L^2 \rightarrow (L')^2 = (1 + \gamma^2\beta^2)L^2 \rightarrow L = L'/\gamma. \quad (22)$$

This calculation demonstrates the correlated nature of the LC and RS effects that follow from the interval transformation equations in (20). It is claimed, in many text books, that the existence of these effects can be deduced from simple inspection of the invariant interval relation (21). On the hypothesis that frames exist where $\Delta t = 0$ and $\Delta t' \neq 0$ it is obvious from (21) that, for such frames, $L \neq L'$. In fact, following Langevin [7], a space-like interval may be defined as one in which a frame exists where $\Delta t = 0$ but none with $\Delta x = 0$; a time-like interval is defined as one in which a frame exists where $\Delta x = 0$, but none with $\Delta t = 0$. The space-like or time-like character of the interval between any two events is then a Lorentz-invariant property. However, the apparent symmetry here between space and time [8] is broken. For event pairs with a time-like interval, frames *always do exist* in which either $\Delta x = 0$ or $\Delta x \neq 0$, whereas due to the universal (spatial coordinate independent) time dilation relation (3) if a frame exists for which Δt vanishes *it must vanish for all inertial frames*. This is quite clear from inspection of Fig. 1. There are therefore two distinct categories of event pairs with space-like intervals, those for which Δt vanishes in all inertial frames and those in which $\Delta t \neq 0$ in all inertial frames.

Inspection of the standard transformation equations in (12) and (17) shows that when $t_i = 0$ and $x_i = 0$ then also $t'_i = 0$ and $x'_i = 0$. The equations then describe events on the worldline of an object which is at the origin, in both S and S' , when $t = t' = 0$. This implies also that $t_0 = 0$ so that the arrays of clocks in S and S' are mutually synchronised when the coordinate origins of S and S' are aligned. In order to now describe the worldline of an object *not* situated at the coordinate origin of S' when the clock arrays have the same mutual synchronisation as above, the transformation equations must be modified.

Making use of the freedom of choice of the coordinate origins, all physical predictions must be unchanged under the coordinate transformations:

$$x_i \rightarrow x_i - x_i(0), \quad x'_i \rightarrow x'_i - x'_i(0), \quad t_i \rightarrow t_i, \quad t'_i \rightarrow t'_i \quad (23)$$

for arbitrary values of the constants $x_i(0)$, $x'_i(0)$. The freedom of choice of the coordinate origin in S' allows the further choice: $x'_i(0) = x_i(0)$. This implies that, as in the standard Lorentz transformation, the coordinate origins in S and S' are aligned when $t = t' = 0$. The transformation equations of events on the worldline of an object at an arbitrary position in the frame S' are then:

$$x'_i(t'_i) - x_i(0) = \gamma[x_i(t_i) - x_i(0) - vt_i] = 0, \quad (24)$$

$$t'_i = \gamma[t_i - v(x_i(t_i) - x_i(0))/c^2]. \quad (25)$$

Note that the space transformation is equivalent to the worldline equations in (1) with $t_0 = 0$ and a particular choice of coordinate origin in S' , and the time transformation equation is the same as Eq. (2) with $t_0 = 0$. When $t_i = 0$ then $t'_i = 0$, independently of the value of $x_i(0)$, i.e. independently of the fixed position of the object in the frame S' . The standard Lorentz transformations are recovered on setting $x_i(0) = 0$. As in the derivation of Eq. (3), elimination of $x_i(t_i)$ from (25), using the last member of (24), gives a spatial-coordinate independent TD relation: $t_i = \gamma t'_i$ (no RS), and the relation $L' = L$ (see Eq. (9)) holds —no LC.

A ‘space-like invariant interval relation’ similar to (21) may be derived from (24) and (25):

$$(\Delta s)^2 = (\Delta x')^2 - c^2(\Delta t')^2 = (\Delta x)^2 - c^2(\Delta t)^2 > 0 \quad (26)$$

where

$$\Delta x' \equiv x'_2(t'_2) - x_2(0) - x'_1(t'_1) + x_1(0) = 0, \quad (27)$$

$$\Delta t' \equiv t'_2 - t'_1, \quad (28)$$

$$\Delta x \equiv x_2(t_2) - x_1(t_1) - (x_2(0) - x_1(0)) = x_2(t_2) - x_1(t_1) - L, \quad (29)$$

$$\Delta t \equiv t_2 - t_1. \quad (30)$$

Setting t_2 equal to t_1 gives $\Delta t = 0$ and $\Delta x = 0$. Since $\Delta x' = 0$ at all times (Eq. (27)), (26) then gives $\Delta t' = 0$, i.e. $t'_2 = t'_1$ — no RS and, unlike (21), no prediction of LC.

To compare the generalised transformation equations of (24) and (25) with the standard ones of Eqs. (12) and (17) it is convenient to write the former as (dropping explicit time arguments)

$$x'_i = \gamma[x_i - vt_i] + X_i, \quad (31)$$

$$t'_i = \gamma[t_i - vx_i/c^2] + T_i \quad (32)$$

so that the generalised transformation equations differ from the standard ones by the inclusion of the additive constants X_i and T_i on their right hand sides where:

$$X_i \equiv x_i(0)(1 - \gamma), \quad (33)$$

$$T_i \equiv \gamma vx_i(0)/c^2. \quad (34)$$

Comparison of (31) and (32) with the standard transformation equations (12) and (17) further elucidates the origin of the spurious RS and LC effects which are derived from the latter equations. On the assumption that x_1 and x_2 are specified in the same coordinate system in the frame S and that t_1 and t_2 are measured by the same network of synchronised clocks in the frame S, the space and time coordinates in the generalised and standard transformation equations are related as follows:

$$x_i = x_i, \quad t_i = t_i, \quad x'_i = x'_i + X_i, \quad t'_i = t'_i + T_i. \quad (35)$$

Thus the coordinate systems in which x'_i and t'_i are defined have coordinate origins displaced by distances X_i and T_i respectively compared to those in which x_i and t_i are defined. It follows from (33), (34) and (35), that

$$x'_2 - x'_1 = L = x'_2 - x'_1 + X_2 - X_1 = x'_2 - x'_1 + (x_2(0) - x_1(0))(1 - \gamma) = x'_2 - x'_1 + L(1 - \gamma) \quad (36)$$

so that

$$x'_2 - x'_1 = L' = \gamma L. \quad (37)$$

Also

$$t'_2 - t'_1 = t'_2 - t'_1 + T_2 - T_1 = t'_2 - t'_1 + \frac{\gamma v(x_2(0) - x_1(0))}{c^2} = t'_2 - t'_1 + \frac{v(x'_2 - x'_1)}{c^2} \quad (38)$$

where, in the last member, (9) and (37) have been used. For simultaneous events in the frame S', $t'_2 = t'_1$, (38) gives:

$$t'_2 - t'_1 = -\frac{v(x'_2 - x'_1)}{c^2} \quad (t'_2 = t'_1) \quad (39)$$

which is just the RS effect of Eq. (19). The origin of this effect and of the correlated LC effect of (37) is then seen to be, as previously pointed out, and now shown explicitly in (35), the use of different i -dependent coordinate systems in the frame S' to specify the events (x'_i, t'_i) .

The necessity to include the additional constants X_i and T_i in the standard Lorentz transformation equations, in order to correctly describe events on the worldlines of objects at different positions, was clearly stated by Einstein in the original special relativity paper [1] just after the derivation of the standard Lorentz transformation equations:

‘Macht man über die Anfangslage des bewegten Systems und über den Nullpunkt von τ keinerlei Voraussetzung, so ist auf den rechten Seiten dieser Gleichungen je eine additive Konstante zuzufügen’

or, in English:

‘If no assumption whatever be made as to the initial position of the moving system and as to the zero point of τ an additive constant is to be placed on the right side of these equations’

The quantity τ is t' in the notation of the present paper.

The arguments presented above suggest that all text books treating special relativity are in error when presenting ‘relativity of simultaneity’ and ‘length contraction’ as *bona fide* physical effects and should be rewritten.

Unlike for the case of time dilation, no experiments sensitive to the existence of putative ‘relativity of simultaneity’ or ‘length contraction’ effects have been performed to date [9].

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