

# Differential equations, Newton's laws of motion and relativity

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## **Abstract**

Adaptation of a calculation due to Lewis allows derivation of all equations of Special Relativity by straightforward application of Newton's Laws of motion and the equivalence of work and energy. How the spurious 'length contraction' and 'relativity of simultaneity' effects of conventional Special Relativity arise from incorrect manipulation of integration constants, in the description of space-time experiments, is explained. Only the universal (position-independent) time dilation effect of Special Relativity distinguishes it from Galilean Relativity.

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# 1 Introduction

The laws of physics are mathematically encapsulated in differential equations [1] but in their application to specific problems it is necessary to find appropriate solutions of these equations. This requires, firstly, the introduction of systems of units in order to parameterise physical quantities, secondly, coordinate axes to specify values of the physical parameters, thirdly, some initial conditions (fixed values of the parameters) to describe the actual physical problem under consideration and, lastly, the integrals of the relevant differential equations. For the mechanical and kinematical problems to be discussed in the present article, all the differential equations concerned are ordinary ones, of first order, with constant coefficients, so all the integrals reduce to the simple form:  $k \int dx = kx + \text{constant}$ . It will be seen that an inappropriate choice of certain constants of integration in problems involving space and time measurements has obfuscated predictions obtained from the Lorentz Transformations (LT) of Special Relativity (SpR) for more than a century now. This situation has been previously discussed in [2, 3, 4, 5] and references therein.

The mathematics employed in the present paper is elementary, however great care is taken throughout to precisely define the operational meaning of all symbols appearing in the equations. It will be seen that it is just the failure to correctly assign the values of fixed parameters specifying the initial conditions of space-time experiments, involving clocks in motion, that underlies the prediction of spurious ‘length contraction’ and ‘relativity of simultaneity’ effects in conventional SpR.

This paper is organised as follows: in the following section spatial transformations in both Galilean relativity (GaR) and SpR are discussed in the context of Newton’s First Law of motion. In Section 3 time dilation and the equivalent interval Lorentz time transformation are derived by posing general ansätze for relativistic momentum and energy and applying Newton’s Second Law of motion as well as the equivalence of work and energy. This calculation, in the guise of a prediction for relativistic mass increase, is due to Lewis [6] and was later given in the Feynman Lectures in Physics [7]. The interval transformation equations obtained in the previous sections are integrated in Section 4 and used to obtain the transformations of event coordinates  $(x,t)$  in a frame S into another inertial frame S’:  $(x'(x,t),t'(x,t))$ . The events considered lie on the worldline of a body that is in motion in the frame S and at rest in S’. In Section 5 event transformation equations for two spatially-separated bodies at rest in S’ are considered. Finally, Section 6 explains the erroneous assignment of coordinate origins that results in ‘length contraction’ and ‘relativity of simultaneity’ in conventional SpR.

## 2 Newton’s First Law and spatial transformations

Newton’s first law of motion [8]:

Every body preserves its state of being at rest or of moving uniformly straight forward except in so far as it is compelled to change its state by forces impressed.

is expressed, mathematically, by the first order differential equation <sup>1</sup>:

$$\frac{d\mathbf{s}}{dt} = \mathbf{v} = \text{constant} \quad (2.1)$$

where  $d\mathbf{s}$  is an infinitesimal spatial displacement of the body during the infinitesimal time interval  $dt$ , and the constant  $\mathbf{v}$  is, by definition, the velocity which quantitatively expresses the state of ‘moving uniformly straight forward’.

Integration of (2.1) gives:

$$\int_{\mathbf{s}(t_0)}^{\mathbf{s}(t)} d\mathbf{s} = \mathbf{v} \int_{t_0}^t dt \quad (2.2)$$

i.e.

$$\mathbf{s}(t) - \mathbf{s}(t_0) = \mathbf{v}(t - t_0)$$

or, equivalently:

$$\mathbf{s}(t) - \mathbf{s}(t_0) - \mathbf{v}(t - t_0) = 0. \quad (2.3)$$

The time interval  $t - t_0$  and the spatial interval  $\mathbf{s}(t) - \mathbf{s}(t_0)$  in (2.3) are, respectively, invariant with respect to the initial setting of the clock used to measure the epoch  $t$  and the coordinate system used to specify the position of the body.

Introducing the coordinate frame S in which the body moves with velocity  $\mathbf{v}$  and the frame S' in which the body is at rest, as well as coordinate axes  $x, x'$  in S, S' parallel to the direction of motion of the body, enables (2.3) to be written ( $v = |\mathbf{v}|$ ):

$$x(t) - x(t_0) - v(t - t_0) = 0 \quad (2.4)$$

while the corresponding relation in the frame S' (where the velocity of the body vanishes) is:

$$x'[t'(t)] - x'[t'(t_0)] = 0. \quad (2.5)$$

The epochs  $t'(t)$  and  $t'(t_0)$  are recorded by a clock at rest in the frame S' while the epoch  $t$  is recorded by a clock at rest in the frame S. The functional dependence  $t'(t)$  will be derived from Newton's Second Law in the following section.

It is illuminating to compare the equations of motion (worldlines) (2.4) and (2.5) where all mathematical symbols have a clear operational definition, with the conventional relativistic spatial transformation laws purporting to relate an event  $(x, t)$  in the frame S with the same event  $(x', t')$  as observed in the frame S':

$$x' = x - vt \quad [\text{Galilean relativity (GaR)}], \quad (2.6)$$

$$x' = \gamma(x - vt) \quad [\text{Special relativity (SpR)}] \quad (2.7)$$

where  $\gamma \equiv 1/\sqrt{1 - (v/c)^2}$  and  $c$  is the speed of light in free space.

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<sup>1</sup>Three-vectors are denoted throughout by boldfaced type, as  $\mathbf{v}, \mathbf{p}, \mathbf{F}$ .

Evidently (2.4) and (2.5) can be combined in a single equation as:

$$x'[t'(t)] - x'[t'(t_0)] = g\{x(t) - x(t_0) - v(t - t_0)\} = 0 \quad (2.8)$$

where  $g$  is an arbitrary, finite, constant or function of  $v$ . The GaR transformation of (2.6) is recovered on setting  $g = 1$ ,  $t_0 = 0$ , and  $x'[t'(t_0)] = x(t_0) = 0$  in this equation to yield

$$x'[t'(t)] = x(t) - vt = 0 \quad (2.9)$$

while setting  $g = \gamma$ ,  $t_0 = 0$ , and  $x'[t'(t_0)] = x(t_0) = 0$  in (2.8) gives

$$x'[t'(t)] = \gamma(x(t) - vt) = 0. \quad (2.10)$$

Consistency of (2.6) with (2.9) and (2.7) with (2.10) then requires that  $x' = 0$  and  $x(t = 0) = 0$  in (2.6) and (2.7). This comparison shows that the transformation formulas (2.6) and (2.7) are only valid for particular choices of clock offset in the frame S and coordinate systems in the frames S and S'. Furthermore the physical meaning of both the GaR and SpR spatial transformations is the same as that of the separate equations of motion (2.4) in S and (2.5) in S'. A corollary of this (since the coefficient  $g$  in (2.8) is arbitrary) is that *there is no physical distinction between the (correctly interpreted) GaR transformation of (2.6) and the SpR transformation of (2.7)*.

### 3 Newton's Second Law and temporal transformations

Newton's Second Law of motion states [9]:

A change of motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.

Newton previously defines 'quantity of motion' [10]:

#### Definition 2

Quantity of motion is a measure of motion that arises from the velocity and the quantity of matter jointly.

as well as 'impressed force' [10]:

#### Definition 4

Impressed force is the action exerted on a body to change its state of either resting or of moving uniformly straight forward.

Newton's definition of 'quantity of matter' somewhat, however, begs the question, since only how to calculate it from the density and volume of a body is stated as well as its proportionality to weight. Following Newton, the 'quantity of matter' associated with a body will be denoted by  $m$  and called 'mass'. This Newtonian concept is a fixed

physical attribute of a body whether in motion or at rest, to be distinguished from the velocity-dependent masses employed in some formulations of SpR. Calling ‘quantity of motion’ (in French, literally, ‘quantité de mouvement’) momentum and denoting it by  $\mathbf{p}$ , Newton’s Definition 2 states that  $\mathbf{p} = m\mathbf{v}$  where the velocity  $\mathbf{v}$  is defined in Eq. (2.1). Newton’s formulation of his Second Law is therefore not (as in elementary text books) ‘force = mass  $\times$  acceleration’ but instead:

$$\frac{d\mathbf{p}}{dt} \equiv \frac{d(m\mathbf{v})}{dt} = \mathbf{F} \quad (3.1)$$

where the symbol  $\mathbf{F}$  denotes the impressed force that is parallel to  $d\mathbf{p}/dt$ . This is a first order differential equation, not a second order one, as in the conventional statement of the Second Law, and is particularly well suited to relativistic generalisation. Indeed, in SpR, the form of the law is unchanged, only the definition of momentum is modified. A general modification of the definition of momentum, consistent with dimensional analysis, is:

$$\mathbf{p} \equiv m\gamma(v)\mathbf{v} = m\gamma(v)\frac{d\mathbf{s}}{dt} \quad (3.2)$$

where  $\gamma(v)$  is an arbitrary real dimensionless function of  $v$ . It will be seen that inserting this generalised definition of momentum in (3.1) allows an unambiguous determination of the function  $\gamma(v)$  as well as suggesting the relativistic generalisation of the energy concept of classical mechanics.

Taking the scalar product of both sides of (3.1) with  $2\mathbf{p}$  and using (3.2) gives:

$$2\mathbf{p} \cdot \frac{d\mathbf{p}}{dt} = 2m\gamma(v)\frac{d\mathbf{s}}{dt} \cdot \mathbf{F} = 2m\gamma(v)\frac{dW}{dt} = 2m\gamma(v)\frac{dE}{dt} \quad (3.3)$$

where the equivalence of mechanical work to energy:

$$dW = \mathbf{F} \cdot d\mathbf{s} = dE \quad (3.4)$$

is assumed. Since the dimensions of energy are  $ML^2T^{-2}$  the last member of (3.3) suggests an expression for the relativistic energy,  $E$  of a body of mass  $m$  and velocity  $v$ :

$$E \equiv V^2m\gamma(v) \quad (3.5)$$

where  $V$  is a universal constant with dimensions of velocity, the physical significance of which will be elucidated below. Multiplying Eq. (3.3) through by the factor  $V^2$  and cancelling out the time differential in the denominator gives the first order differential equation ( $p \equiv |\mathbf{p}|$ ):

$$2V^2\mathbf{p} \cdot d\mathbf{p} = V^2d(p^2) = 2EdE = d(E^2) \quad (3.6)$$

which yields, on integration:

$$V^2p^2 = E^2 + C. \quad (3.7)$$

The integration constant  $C$  is determined by setting  $v = 0$ ,  $p = 0$ , which, with the definition (3.5) of  $E$  gives:

$$0 = V^4m^2\gamma(0)^2 + C \quad (3.8)$$

so that

$$E^2 = V^4m^2\gamma(0)^2 + V^2p^2. \quad (3.9)$$

Substituting for  $p$  and  $E$  in this equation using the definitions (3.2) and (3.5), cancelling out a common factor  $m^2$ , rearranging, and taking a square root, it is found that:

$$\gamma(v) = \frac{\gamma(0)}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} \quad (3.10)$$

where it is assumed that  $\gamma(0) = 1$  in order to recover the Newtonian definition of momentum  $\mathbf{p} = m\mathbf{v}$  in the limit  $v \ll V$ .

Taking the ratio of Eq. (3.2) to (3.5) and using (3.9) with  $\gamma(0) = 1$  it is found that:

$$v = \frac{pV^2}{E} = \frac{pV}{\sqrt{m^2V^2 + p^2}}. \quad (3.11)$$

Inspection of this equation gives one important physical meaning of the universal constant  $V$ . In any inertial frame where the relativistic energy,  $E$ , is much greater than the energy equivalent,  $mV^2$ , of the mass of the body, the velocity of the body will be less than, but very close to,  $V$ . For a massless particle, such as a photon, (3.11) gives exactly  $v = V$  for all values of  $E$  and  $p = E/V$ . In this way  $V$  may be identified with the speed of light in free space,  $c$  [11].

Combining (3.5) and (3.10) it can be seen (as first noted by Einstein [12]) that  $V$  is also the limiting velocity of any body, due to the action of impressed forces, in any inertial frame. In the limit  $v \rightarrow V$ , (3.10) shows that  $E \rightarrow \infty$ , requiring the application of an infinite amount of mechanical work to attain the speed  $V$ .

Setting  $\gamma(0) = 1$  in (3.9), transposing and inserting for  $p$  and  $E$  from (3.2) and (3.5) gives:

$$E^2 - V^2p^2 = V^4m^2 = V^4\gamma(v)^2m^2 - V^2\gamma(v)^2v^2m^2. \quad (3.12)$$

Because the first member of (3.12) is independent of  $v$  the relation holds in all inertial frames i.e. for all values of  $v$ ,  $E$  and  $p$  consistent with Eqs. (3.2) and (3.5), including  $v = 0$ ,  $E = mV^2$  and  $p = 0$ . Cancelling a common factor of  $m^2V^2$ , setting  $v = ds/dt$ , multiplying through by  $(dt)^2$  and dividing through by  $\gamma(v)^2$  yields the time-like invariant interval relation:

$$V^2(dt)^2 - (ds)^2 = \left(\frac{Vdt}{\gamma(v)}\right)^2 \equiv V^2(d\tau)^2. \quad (3.13)$$

Setting  $ds = 0$ ,  $v = 0$  and  $\gamma(v) = 1$  it is seen that  $dt \rightarrow d\tau = dt'$ , the interval  $dt'$  being that measured by a clock in the rest frame of the body, so that a time dilation relation between the time intervals  $dt$  and  $dt' = d\tau$  is established by the last member of (3.13):

$$dt = \gamma(v)dt' = \gamma(v)d\tau. \quad (3.14)$$

Combining this equation with the equation of motion of the body in the frame S:  $ds = vdt$  yields the Lorentz transformation for time intervals:

$$dt' = \frac{dt}{\gamma(v)} = \frac{\gamma(v)dt}{\gamma(v)^2} = \gamma(v) \left[1 - \left(\frac{v}{V}\right)^2\right] dt = \gamma(v) \left[dt - \frac{vds}{V^2}\right]. \quad (3.15)$$

Note that, given the equation of motion in the frame S, the time dilation relation (3.14) and the Lorentz transformation of time, (3.15), are physically equivalent.

To summarize, the coordinate-independent space-time transformation equations of infinitesimal spatial and temporal intervals are therefore, in GaR:

$$ds' = ds - vdt = 0, \quad dt' = dt \quad (3.16)$$

and in SpR:

$$ds' = \gamma(v)(ds - vdt) = 0, \quad dt' = \gamma(v) \left[ dt - \frac{vds}{V^2} \right] \quad (3.17)$$

or, more simply:

$$ds' = 0, \quad ds = vdt, \quad dt = \gamma(v)dt'. \quad (3.18)$$

Since the factor  $\gamma(v)$  in the space transformation equation in (3.17) may be replaced by an arbitrary, finite, constant or function of  $v$ , without changing the physical meaning of the equation, the space transformations of SpR and GaR are physically equivalent. The only difference between GaR and SpR is the replacement of universal time intervals  $dt = dt'$  of GaR by the time dilation relation  $dt = \gamma(v)dt'$  of SpR which implies that a clock, at rest in the frame S' considered above, will be seen to be running slow, relative to an identical clock at rest in the frame S, by all observers.

## 4 Transformations of space and time coordinates between inertial frames

To obtain the transformation equations between an 'event'  $(x,t)$  in the frame S and the corresponding one  $(x'(x,t),t'(x,t))$  in the frame S', as specified by the above coordinate systems, the interval transformations of Eq. (3.16) or (3.17) must be integrated. Performing the integration gives the indefinite integrals:

$$x' = x - vt + X^{\text{GaR}}, \quad t' = t + T^{\text{GaR}} \quad [\text{Galilean relativity}] \quad (4.1)$$

$$x' = \gamma(v)[x - vt] + X^{\text{SpR}}, \quad t' = \gamma(v) \left[ t - \frac{vx}{V^2} \right] + T^{\text{SpR}} \quad [\text{Special relativity}] \quad (4.2)$$

The values of the integration constants:  $X^{\text{GaR}}$ ,  $T^{\text{GaR}}$ ,  $X^{\text{SpR}}$  and  $T^{\text{SpR}}$  depend on the choice of coordinate origins and clock offsets in the frames S and S'. Allowing a completely arbitrary choice of these constant parameters the above equations may be written, for GaR as:

$$x'(t) - x'(t_0) = x(t) - x(t_0) - v(t - t_0) = 0, \quad (4.3)$$

$$t'(t) - t'(t_0) = t - t_0 \quad (4.4)$$

which give:

$$X^{\text{GaR}} = x'(t'_0) - x(t_0) + vt_0, \quad T^{\text{GaR}} = t'_0 - t_0 \quad (4.5)$$

and for SpR as:

$$x'(t) - x'(t_0) = \gamma(v)[x(t) - x(t_0) - v(t - t_0)] = 0, \quad (4.6)$$

$$t'(t) - t'(t_0) = \gamma(v) \left[ t - t_0 - \frac{v[x(t) - x(t_0)]}{V^2} \right] \quad (4.7)$$

so that

$$X^{\text{SpR}} = \gamma(v)[vt_0 - x(t_0)] + x'(t'_0), \quad T^{\text{SpR}} = \gamma(v) \left[ \frac{vx(t_0)}{V^2} - t_0 \right] + t'_0. \quad (4.8)$$

Choosing now  $t_0 = t'(t_0) = 0$  and, making use of the freedom of choice of coordinate origins in the frames S and S', setting  $x'(t' = 0) = x(t = 0) = \chi$ , gives for GaR:

$$x'(t') - \chi = x(t) - \chi - vt = 0, \quad t' = t. \quad (4.9)$$

i.e.

$$X^{\text{GaR}} = 0, \quad T^{\text{GaR}} = 0. \quad (4.10)$$

and for SpR:

$$x'(t') - \chi = \gamma(v)[x(t) - \chi - vt] = 0, \quad (4.11)$$

$$t'(t) = \gamma(v) \left[ t - \frac{v[x(t) - \chi]}{V^2} \right] = \frac{t}{\gamma(v)} \quad (4.12)$$

so that

$$X^{\text{SpR}} = [1 - \gamma(v)]\chi, \quad T^{\text{SpR}} = \frac{\gamma(v)v\chi}{V^2} \quad (4.13)$$

which are physically equivalent to:

$$x'(t') = \chi, \quad x(t) = \chi + vt, \quad t'(t) = \frac{t}{\gamma(v)}. \quad (4.14)$$

The first two relations in (4.14) are solutions of the differential equations of motion of the body in the frames S' and S respectively, for a particular choice of coordinate systems and clock offsets, which are identical in GaR and SpR. The time dilation relation  $t = \gamma(v)t'$ , relating the clock epochs  $t$  and  $t'$ , is therefore the unique physical effect that distinguishes SpR from GaR.

As discussed in detail elsewhere [3], the necessity to include the additive constants  $X^{\text{SpR}}$  and  $T^{\text{SpR}}$  on the right sides of the 'standard' LT:  $x' = \gamma(x - vt)$ ,  $t' = \gamma(t - vx/c^2)$  in order to correctly describe a synchronised clock not placed at the origin of coordinates in S', was clearly stated in Einstein's original paper on SpR [12]. However, this was never done by him nor, to the present writer's best knowledge, by any other author before the work presented in Ref. [2].

## 5 Transformation equations for two spatially-separated bodies at rest in the same inertial frame

Using the same coordinate systems as in (4.14), the transformation equations for two clocks  $C'_1$  and  $C'_2$ , recording epochs  $t'_1$  and  $t'_2$  respectively, placed at arbitrary positions on the  $x'$  axis, with arbitrary epoch offsets,  $t'_1(0)$  and  $t'_2(0)$ , are ( $\chi_2 > \chi_1$ ):

$$x'_1(t'_1) = \chi_1, \quad x_1(t_1) = \chi_1 + vt_1, \quad t'_1(t_1) - t'_1(0) = \frac{t_1}{\gamma(v)}, \quad (5.1)$$

$$x'_2(t'_2) = \chi_2, \quad x_2(t_2) = \chi_2 + vt_2, \quad t'_2(t_2) - t'_2(0) = \frac{t_2}{\gamma(v)}. \quad (5.2)$$



It follows from (5.1) and (5.2) that

$$\chi_2 - \chi_1 = x_2(t_2) - x_1(t_1) - v(t_2 - t_1). \quad (5.3)$$

The separation of the clocks at a fixed epoch in the frame S:  $t_1 = t_2 = t$  is then:

$$L \equiv x_2(t) - x_1(t) = \chi_2 - \chi_1 = x'_2(t'_2) - x'_1(t'_1) \equiv L'. \quad (5.4)$$

There is therefore no ‘length contraction’ effect when the separation of the moving clocks is measured, at any epoch,  $t$ , in the frame S [3, 4, 5].

It is important to note that the equality of length intervals:  $L' = L$  is not an artifact of the particular choice of coordinate origins assumed in (5.1) and (5.2). With the same separation between the clocks, but a different choice of coordinate origin in the frame S':  $x' \rightarrow x' + D'$  (i.e. shifting the origin of coordinates by the distance  $D'$  in the negative  $x'$  direction) modifies the first equations in (5.1), (5.2) to  $x'_1(t'_1) = \tilde{\chi}_1$ ,  $x'_2(t'_2) = \tilde{\chi}_2$  where  $\tilde{\chi}_1 \equiv \chi_1 + D'$ ,  $\tilde{\chi}_2 \equiv \chi_2 + D'$ . However the spatial separation does not depend on the value of  $D'$ :

$$\tilde{L}' \equiv \tilde{\chi}_2 - \tilde{\chi}_1 = (\chi_2 + D') - (\chi_1 + D') = \chi_2 - \chi_1 = L'. \quad (5.5)$$

This is a consequence of the translational invariance of physical quantities in flat space [4]. A similar argument applied in the frame S shows that the separation  $L$  is also independent of the choice of coordinate origin in this frame.

If the clocks  $C'_1$  and  $C'_2$  are synchronised so that  $t'_1(0) = t'_2(0) = 0$  when  $t = 0$  the time dilation relations in (5.1) and (5.2) become, respectively,  $t'_1 = t_1/\gamma(v)$ ,  $t'_2 = t_2/\gamma(v)$ , so that the clocks remain synchronised at any epoch  $t$  in the frame S:

$$t'_1(t) = \frac{t}{\gamma(v)} = t'_2(t) \quad (5.6)$$

so there is no ‘relativity of simultaneity’ effect for spatially-separated, synchronised, clocks [3, 5]. How the spurious ‘length contraction’ and ‘relativity of simultaneity’ effects of conventional SpR arise from errors in setting parameters specifying initial conditions in the application of Lorentz transformations for events is explained in the following section.

## 6 The ‘length contraction’ and ‘relativity of simultaneity’ effects of conventional Special relativity

If the clock  $C'_1$  is placed at the origin of coordinates in S' so that  $\chi_1 = 0$  the corresponding coordinate transformations are given by (4.11) and (4.12) as:

$$x'_1(t'_1) = \gamma(v)[x_1(t_1) - vt_1] = 0, \quad (6.1)$$

$$t'_1(t_1) = \gamma(v) \left[ t_1 - \frac{vx_1(t_1)}{V^2} \right]. \quad (6.2)$$

The transformation equations for the clock  $C'_2$  placed at  $x'_2(t'_2) = \chi_2 = \chi_2 - \chi_1 \equiv L$  are given, by the same equations, as:

$$x'_2(t'_2) - L = \gamma(v)[x_2(t_2) - L - vt_2] = 0, \quad (6.3)$$

$$t'_2(t_2) = \gamma(v) \left[ t_2 - \frac{v(x_2(t_2) - L)}{V^2} \right]. \quad (6.4)$$

$$L' = \gamma L \quad \gamma = 2 \quad t_1 = t_2 = 0$$

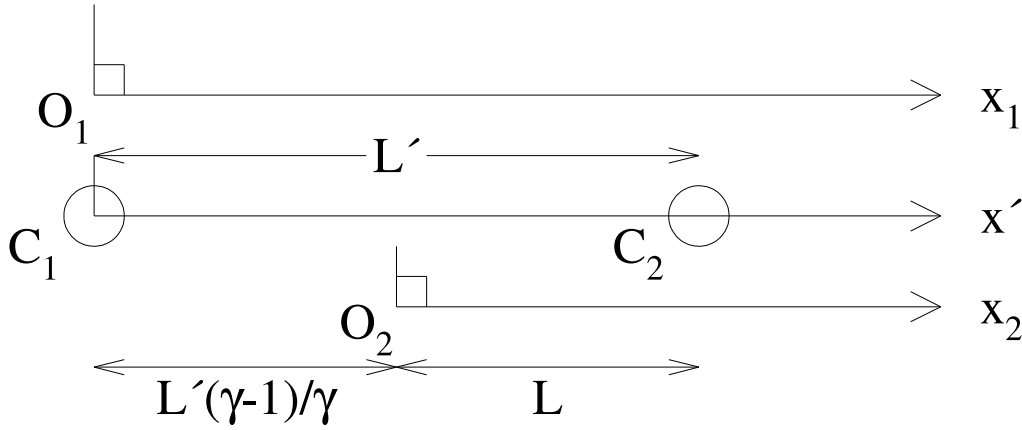


Figure 1: Clock  $C_1$  is placed at the origin of coordinates in the frame  $S'$  and the clock  $C_2$  at  $x' = L'$  in this frame. The conventional space Lorentz transformation  $x' = \gamma(x - vt)$  applied at  $t = 0$  to  $C_1$  and  $C_2$ , respectively, give  $x_1 = 0$  and  $x_2 = L'/\gamma$ . As shown, the origin,  $O_2$ , of the coordinate system used to specify the position of  $C_2$  in the frame  $S$  is then shifted by the distance  $L'(\gamma - 1)/\gamma$  from the origin  $O_1$  of the coordinate system used to specify the position of  $C_1$  on  $S$ . This use of different coordinate systems in the frame  $S$  for  $C_1$  and  $C_2$  is the origin of the spurious 'length contraction' effect:  $L = L'/\gamma$  of conventional SpR.

The 'length contraction' and 'relativity of simultaneity' effects arise from the erroneous assumption that the Lorentz transformations in the 'standard' form of (6.1) and (6.2) (omitting however the crucial '= 0' on the right side) are directly applicable to the event coordinates of the clock  $C_2$ . i.e. instead of (6.3) and (6.4) it is assumed that:

$$x'_2(t'_2) = \gamma(v)[x_2(t_2) - vt_2] = L' \neq 0, \quad (6.5)$$

$$t'_2(t_2) = \gamma(v) \left[ t_2 - \frac{vx_2(t_2)}{V^2} \right]. \quad (6.6)$$

Setting  $t_1 = t_2 = t$  and combining (6.1) and (6.5) it is found that:

$$L' \equiv x'_2(t'_2) - x'_1(t'_1) = \gamma(v)[x_2(t) - x_1(t)] \equiv \gamma(v)L. \quad (6.7)$$

This is the putative 'length contraction' (LC) effect:  $L = L'/\gamma(v)$ .

Combining (6.2) and (6.6) when  $t_1 = t_2 = t$  it is found that:

$$t'_2(t) - t'_1(t) = \frac{\gamma(v)v[x_1(t) - x_2(t)]}{V^2} = -\frac{\gamma(v)vL}{V^2} \quad (6.8)$$

which gives a 'relativity of simultaneity' (RS) effect since  $t'_2 - t'_1 \neq 0$  when  $t_1 = t_2$ . The above is the standard text book derivation of LC and RS. The derivation of (6.7) (but not of (6.8)) was given by Einstein in the first SpR paper [12].

Subtracting  $L$  from both sides of (6.5) and rearranging, using the relation  $L' = \gamma(v)L$ , it may be written in a manner which facilitates comparison with Eq. (6.3), in which the

operational meaning of all symbols is clearly defined. This gives, using (6.7):

$$x'_2(t'_2) - L' = \gamma(v)[x_2(t_2) - L - vt_2] = 0. \quad (6.9)$$

As is clear from a comparison of this equation with (6.3) and inspection of Fig. 1, this corresponds to a *different choice of coordinate system in the frame S to specify the position of the clock C'\_2 to that used, in the same frame, to specify the position of the clock C'\_1*. In fact, the origin of the coordinate system used for C'\_2 is displaced from that of the system used for C'\_1 by the distance (see Fig. 1)  $L'(\gamma(v) - 1)/\gamma(v)$  in the positive  $x$ -direction.

Comparing (6.6) with (6.4), which correctly describes synchronisation of C'\_1 and C'\_2, ((6.1)-(6.4) give  $t'_1 = t'_2 = 0$  when  $t_1 = t_2 = 0$ ) shows that (6.6) and (6.8) are corrected by the replacement:  $x_2(t) \rightarrow x_2(t) - L$  so that (6.8) becomes:

$$t'_2(t) - t'_1(t) = \frac{\gamma(v)v[x_1(t) - x_2(t) + L]}{V^2} = \frac{\gamma(v)}{V^2}[-L + L] = 0 \quad (6.10)$$

demonstrating the spurious nature of the demonstration of RS derived from Eqs. (6.2) and (6.6). For direct comparison of the 'standard' time transformation equation (6.6) with (6.4) the former may be written as:

$$t'_2(t_2) - t'_2(0) = \gamma(v) \left[ t_2 - \frac{v[x_2(t_2) - L]}{V^2} \right] \quad (6.11)$$

where  $t'_2(0) \equiv \gamma(v)vL/V^2$ . Setting  $t_1 = t_2 = 0$  in (6.1)-(6.4) gives  $t'_1 = t'_2 = 0$  whereas the same settings in (6.1) and (6.11) give, respectively,  $t'_1 = 0$  and  $t'_2 = t'_2(0) = \gamma(v)vL/V^2$ . This shows that the putative RS effect *simply results from the use of clocks that are, by construction of the transformation equations, unsynchronised*. Clearly, synchronisation of the settings of two clocks, at any chosen instant, is a purely mechanical or electronic procedure, under the complete control of the experimenter, without any physical significance. The arguments just presented show that the LC and RS effects result from an inconsistent assignment of coordinate systems and clock settings for two spatially separated bodies and so are not *bona fide* predictions of SpR.

# References

- [1] R.B.Lindsay and H.Margenau ‘Foundations of Physics’ (Dover, New York, 1957) Section 1.7, p.29.
- [2] J.H.Field, ‘The Local Space-Time Lorentz Transformation: a New Formulation of Special Relativity Compatible with Translational Invariance’, arXiv pre-print: <http://xxx.lanl.gov/abs/physics/0501043v3>. Cited 30 Nov 2007.
- [3] J.H. Field, *Fundamental Journal of Modern Physics*, Vol **2** Issue 2 139 (2011); arXiv pre-print <http://xxx.lanl.gov/abs/1210.2270v1>. Cited 8 Oct 2012.
- [4] J.H. Field, *Fundamental Journal of Modern Physics*, Vol **4** Issue 1-2 1 (2012); arXiv pre-print: <http://xxx.lanl.gov/abs/1307.7962>. Cited 30 Jul 2013.
- [5] J.H. Field, *Fundamental Journal of Modern Physics*, Vol **6** Issue 1-2 1 (2013). arXiv pre-print: <http://xxx.lanl.gov/abs/1405.5174>. Cited 23 Apr 2014.
- [6] G.N.Lewis, *Phil. Mag.* **56** Vol. 16 No. (5 (1908)).
- [7] R.P.Feynman, R.B.Leighton and M.Sands, ‘Lectures on Physics, Vol I (Addison-Wesley, Reading, Massachusetts, 1964) Section 15-9.
- [8] Sir Isaac Newton, ‘Philosophie Naturalis Principia Mathematica’, Trans. J.B.Cohen and Anne Whitman (U.Cal. Press, Berkeley CA, 1999) p.416.
- [9] Ref. [8] p.404.
- [10] Ref. [8] p.405.
- [11] J.H.Field, *Helv. Phys. Acta* **70** 542 (1997), arXiv pre-print: <http://xxx.lanl.gov/abs/physics/0410262v1>. Cited 27 Oct 2004.
- [12] A. Einstein, *Annalen der Physik* **17**, 891 (1905). English translation by W.Perrett and G.B.Jeffery in ‘The Principle of Relativity’ (Dover, New York, 1952) p.37, or in ‘Einstein’s Miraculous Year’ (Princeton University Press, Princeton, New Jersey, 1998) p.123.